Maximal $\nu_e \to \nu_s$ solution to the solar neutrino problem: just-so, MSW or energy independent?

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Abstract

We examine the maximal $\nu_e \to \nu_s$ solution to the solar neutrino problem. This solution can be motivated by the exact parity model and other theories. The ν_e survival probability exhibits one of three qualitatively different behaviours depending on the value of Δm^2, viz. approximately energy independent, just-so or MSW. By the last of these we mean an enhanced night-time event rate due to regeneration in the Earth. We study all of these possibilities in the context of the recent SuperKamiokande data.

I. INTRODUCTION

Five experiments have measured solar neutrino fluxes that are significantly deficient relative to standard solar model expectations [1,2]. Four out of the five experiments find overall fluxes that are roughly 50% of the theoretical predictions. (The Chlorine experiment sees a larger deficit.) Maximal mixing between the electron neutrino and a sterile flavour has been proposed as the underlying explanation for these observations [3]. This oscillation mode produces a 50% reduction in the day-time solar neutrino flux for a large range of the relevant Δm^2 parameter:

$$10^{-3} > \frac{\Delta m^2}{\text{eV}^2} \gtrsim \text{ few } \times 10^{-10}. \tag{1}$$

The upper bound arises from the lack of $\overline{\nu}_e$ disappearence in the CHOOZ experiment [4][1], while the lower bound is a rough estimate of the transition region between the totally averaged oscillation regime and the "just-so" regime.

The very special feature of maximal mixing between the ν_e and a sterile flavour is well motivated by the Exact Parity Model (also known as the mirror matter model) [3,6]. In this theory, the sterile flavour maximally mixing with the ν_e is identified with the mirror electron neutrino. The characteristic maximal mixing feature occurs because of the underlying exact parity symmetry between the ordinary and mirror sectors. The potentially maximal mixing observed for atmospheric muon neutrinos is beautifully in accord with this framework [7], which sees the atmospheric neutrino problem resolved through '$\nu_\mu \to$ mirror partner' oscillations.[2] *The Exact Parity Model therefore provides a unified understanding of the solar and atmospheric neutrino problems: each is due to maximal oscillations of the relevant ordinary neutrino into its mirror partner.* Note that the mirror neutrino scenario is phenomenologically similar to the pseudo-Dirac scenario [8].

The chlorine result is not quantitatively consistent with this view of the origin of the solar neutrino anomaly, it being about 30% too low. We await with interest some new experiments that have the capacity to double-check this result.

The purpose of this paper is to make a more detailed analysis of the maximal $\nu_e \to \nu_s$ solution to the solar neutrino problem. We do so because of two recent developments: (i) the clarification from Guth et al. [9] that a day-night asymmetry generically exists even for maximal mixing (due to Earth regeneration which affects the night-time events) and (ii) the observation by SuperKamiokande of an interesting feature in the recoil electron energy spectrum for $E > 13$ MeV [2]. We will calculate the day-night asymmetry and the recoil electron spectrum as a function of Δm^2 in the range 10^{-3} eV^2 to 10^{-11} eV^2. We will draw the important conclusion that the maximal $\nu_e \to \nu_s$ scenario has a larger number of characteristic and testable features than realised hitherto. We will summarise the "smoking gun" experimental signatures for this scenario in the concluding section.

II. DAY-NIGHT ASYMMETRY

Guth et al. [9] have recently provided a very lucid account of the physics of the day-night effect for maximally mixed solar neutrinos. This is important for the maximal $\nu_e \to \nu_s$ scenario for two reasons. First, high statistics experiments such as SuperKamiokande have an on-going programme to measure the solar neutrino day-night asymmetry. It had been previously and erroneously thought that maximally mixed ν_e's would *not* give rise to a day-night asymmetry. We will calculate this asymmetry for the total, energy-integrated flux relevant for SuperKamiokande as a function of Δm^2. We will see that the present

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[1]Note that this entire range for Δm^2 does not necessarily lead to any inconsistency with bounds imposed by big bang nucleosynthesis [5].

[2]The mirror partners of the three ordinary neutrino flavours are distinct, effectively sterile light neutrino flavours.

data already rule out a range of Δm^2. If a nonzero asymmetry were to be experimentally established in the future, then this would *not* falsify the maximal $\nu_e \to \nu_s$ scenario, contrary to previous belief. Rather, such an observation would help to pin down the actual value of Δm^2.

The second consequence of doing the physics correctly is the revelation that the night-time flux reduction is generically *not* an energy-independent factor of a half as previously advertised. This is simply because the Earth regeneration effect [14] is energy-dependent. Data samples which do not separate day-time and night-time observations would thus be expected to show a (weak) energy dependent suppression if the maximal $\nu_e \to \nu_s$ scenario is correct.

Since the paper by Guth et al. provides a complete account of how the day-night asymmetry is calculated, we will not repeat all of this material here. Suffice it to say that we checked our numerical procedure by recalculating some of the results given by Guth et al. for maximal $\nu_e \to \nu_\mu$ (or ν_τ) oscillations. We found agreement. The new computation we present here is very similar, but involves changing the effective potential for neutrino oscillations in the Earth to the one relevant for $\nu_e \to \nu_s$ oscillations. It is given by

$$V = \sqrt{2} G_F (N_e - \frac{N_n}{2}) \simeq \frac{G_F}{\sqrt{2}} \frac{\rho}{m_N} (3Y_e - 1), \tag{2}$$

where G_F is the Fermi constant, N_e (N_n) is the terrestrial electron (neutron) number density, m_N is the nucleon mass, ρ is the terrestrial mass density, and Y_e is the number density of electrons per nucleon. We used the terrestrial density profile given in Ref. [10]. The core of the computation is the determination of the recoil electron flux $g(\alpha, T)$ at apparent recoil energy T and zenith angle α (using the notation of Guth et al.). It is given by

$$g(\alpha, T) = \int_0^\infty dE_\nu \Phi(E_\nu) \int_0^{T'_{\max}} dT' R(T, T') \frac{d\sigma}{dT'} (T', E_\nu, \alpha) \tag{3}$$

where $\Phi(E_\nu)$ is the Boron neutrino spectrum [11], $R(T, T')$ is the energy resolution function given by [13]

$$R(T, T') = \frac{1}{\Delta_{T'} \sqrt{2\pi}} \exp \left[-\frac{(T' - T)^2}{2\Delta_{T'}^2} \right], \tag{4}$$

and the effective cross-section is given by [12]

$$\frac{d\sigma}{dT'} (T', E_\nu, \alpha) = P(E_\nu, \alpha) \frac{d\sigma_{\nu_e}}{dT'} (T', E_\nu). \tag{5}$$

The function P is the electron neutrino survival probability incorporating matter effects in the Earth, and $d\sigma_{\nu_e}/dT'$ is the $\nu_e - e$ scattering cross-section. The resolution width is given by [15] $\Delta_{T'}/\mathrm{MeV} \simeq 0.47\sqrt{T'/\mathrm{MeV}}$.

Our result is presented in Fig.1, where the energy integrated day-night asymmetry A_{n-d} is plotted as a function of Δm^2 for maximal $\nu_e \to \nu_s$ oscillations. The asymmetry is defined by

$$A_{n-d} \equiv \frac{\int_{6.5 \text{ MeV}}^\infty dT \, [N(T) - D(T)]}{\int_{6.5 \text{ MeV}}^\infty dT \, [N(T) + D(T)]}, \tag{6}$$

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where 6.5 MeV is the energy threshold relevant for the day-night asymmetry measurement of SuperKamiokande, $D(T)$ is the day-time recoil electron flux and $N(T)$ is the night-time flux. The day-night asymmetry is computed from $g(\alpha, T)$ via the procedure described in Appendix B of Guth et al. An average is performed over all zenith angles and seasons of the year. Observe that the day-night asymmetry is positive and peaks with a value of about 20% when $\Delta m^2 \simeq 10^{-6}$ eV^2. The present SuperKamiokande result [2],

$$A_{n-d} = +0.026 \pm 0.021 \text{ (stat. + sys.)}, \tag{7}$$

yields a 2σ upper bound of roughly $A_{n-d} < 0.068$ which is plotted as a horizontal line in Fig.1. We see that the range

$$2 \times 10^{-7} \lesssim \frac{\Delta m^2}{\text{eV}^2} \lesssim 8 \times 10^{-6} \tag{8}$$

is disfavoured at the 2σ level. The regions immediately to the side of the disfavoured region will obviously be probed as more data are gathered. The asymmetry falls to the 1% level at about $\Delta m^2 = 3 \times 10^{-8}$ eV^2 and 5×10^{-5} eV^2.

If a positive nonzero value for A_{n-d} were to be measured, then an ambiguity would remain in the determination of Δm^2: values on either side of the presently disfavoured region can produce the same A_{n-d}. This ambiguity could in principle be resolved from a determination of the energy dependence of the night-time rate. Figures 2a and 2b depict the energy dependence of the flux reduction for two representative values of Δm^2 on either side of the disfavoured region. Figure 2a takes $\Delta m^2 \simeq 10^{-7}$ eV^2, with the solid (dotted) line showing the ratio of night-time (day-time) flux per unit energy to the no-oscillation expectation. The dot-dashed curve is the average. Note that the day-time rate is rigorously an energy-independent factor of two less than the no-oscillation rate, while the night-time rate is $5 - 8\%$ higher than the day-time rate and is weakly energy-dependent. Figure 2b considers the same quantities for $\Delta m^2 = 10^{-5}$ eV^2. Note that the slopes of the night-time fluxes have opposite signs on opposite sides of the disfavoured region. The energy dependence becomes unobservably small for Δm^2 values away from the interval around 10^{-6} eV^2.

III. RECOIL ELECTRON SPECTRUM

An interesting situation exists at present with regard to the recoil electron energy spectrum measured by SuperKamiokande [2]. If the *hep* neutrino flux predictions from standard solar models are taken at face value, then SuperKamiokande has evidence for a distortion in the boron neutrino induced recoil energy spectrum for energies greater than about 13 MeV. We will call this the "spectral anomaly". More specifically, the spectral anomaly is an excess of events relative to what would be expected on the basis of an energy-independent boron neutrino flux reduction of about 50%. The observed distortion also disfavours the popular small and large mixing angle MSW solutions to the solar neutrino problem.

One can readily identify three possible interpretations of the spectral anomaly: (i) Standard solar models grossly underestimate the *hep* neutrino flux. (ii) There is an as yet unidentified systematic error in the energy resolution function used by SuperKamiokande,

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and/or in their energy calibration. (iii) It is a statistical fluctuation. (iv) New physics is the cause. We will not consider (i) in this paper, and instead focus on (iv). Before doing so, we briefly discuss (ii) as a cautionary note.

Figure 3 illustrates the effect of varying the resolution width $\Delta_{T'}$. We fit the data by minimising the χ^2 function

$$\chi^2 = \sum_{i=1}^{18} \left[\frac{N_i^{\text{exp}} - 0.5 f N_i^{\text{th}}}{\sigma(N_i^{\text{exp}})} \right]^2 + \left[\frac{f-1}{\sigma(f)} \right]^2, \tag{9}$$

where N_i^{exp} is the measured recoil electron flux in energy bin i, N_i^{th} is the theoretical no-oscillation expectation for same, 0.5 represents an energy independent 50% suppression due (for instance) to averaged maximal $\nu_e \to \nu_s$ oscillations and f is a boron neutrino flux normalisation parameter to take account of the theoretical uncertainty $\sigma(f) \simeq 0.19$ in this quantity [11]. We include the two low energy bins, $5.5 - 6.0$ MeV and $6.0 - 6.5$ MeV, as well as the 16 other energy bins used by SuperKamiokande [2]. Seasonal and daily averages are taken. These data are fitted by varying f and the quantity Δ defined through

$$\Delta_{T'} = (\Delta \text{ MeV}) \sqrt{\frac{T'}{\text{MeV}}}. \tag{10}$$

We find the minimum at $f = 0.90$ and $\Delta \simeq 0.51$, with $\chi^2_{\text{min}} \simeq 20$ for $19 - 2 = 17$ degrees of freedom, which is a good fit. The spectral anomaly becomes an artifact of the finite resolution of the detector. SuperKamiokande quote a central value for Δ of about 0.47 [13]. We can see from Fig.3 that a $5 - 10\%$ systematic shift upward of Δ would remove the anomaly. It is interesting to compare this result with Fig.1 from Ref. [16] which shows the effect of an unidentified systematic error in the energy calibration of the detector. Systematic errors in either or both of the energy resolution and energy calibration can in principle explain the spectral anomaly. Having sounded this cautionary note, we now proceed on the basis that SuperKamiokande have in fact correctly determined their energy resolution and calibration capabilities, and that new physics is behind the spectral anomaly.

It is easy to understand how the spectral anomaly can be explained in a very amusing way through maximal $\nu_e \to \nu_s$ oscillations. It could be that Δm^2 is such that the transition energy between the averaged oscillation regime and the just-so regime is about 13 MeV [17]! The spectral anomaly might represent the onset of just-so behaviour. This would of course be a great piece of luck, but we probably should not disregard the possibility just out of world weary pessimism. To determine the Δm^2 value required to achieve this effect, the appropriate χ^2 function is minimised by varying Δm^2 and f. The appropriate χ^2 is similar to Eq.(9), except that $0.5 N_i^{\text{th}}$ is replaced by a properly computed convolution integral. The Δm^2 dependence of χ^2 is shown in Fig.4. There are deep local minima in the 4×10^{-10} eV^2 to 10^{-9} eV^2 region, which then trail off into a flat χ^2 as the averaged oscillation regime is entered. The minimum χ^2 value is about 17 at $\Delta m^2 \simeq 6 \times 10^{-10}$ eV^2 (and $f = 0.92$), which for 17 degrees of freedom represents a excellent fit. This Δm^2 can in principle be further probed through the anomalous seasonal effect. Preliminary calculations employing the Δm^2 which minimises χ^2 show that an effect of about 6% in magnitude can be obtained for the near-far asymmetry for certain of the higher energy bins of SuperKamiokande. We hope to return to an analysis of the seasonal effect in a later paper.

IV. CONCLUSION

Maximal $\nu_e \rightarrow \nu_s$ oscillations have been proposed as a solution to the solar neutrino problem [3,8]. This idea can be well-motivated by the mirror matter hypothesis [3], or by the pseudo-Dirac neutrino idea [8]. In this paper we have demonstrated the following points:

1. Across the allowed Δm^2 spectrum, maximal $\nu_e \rightarrow \nu_s$ oscillations lead to three qualitatively different behaviours for the ν_e survival probability; just-so, MSW or approximately energy independent. These occur in the $\Delta m^2/\text{eV}^2$ ranges $10^{-3} >$ energy independent $>$ few $\times 10^{-5} >$ MSW $>$ few $\times 10^{-8} >$ energy independent $>$ $10^{-9} >$ just-so $>$ few $\times 10^{-10}$. The day-night asymmetry observable provides for an experimental probe in the MSW regime. The recoil electron energy spectrum and the anomalous seasonal effect likewise provide a probe in the just-so regime.

2. Day-night asymmetry data rule out the range $2 \times 10^{-7} - 8 \times 10^{-6}$ eV^2 for Δm^2 at the 2σ level. A positive measurement of the day-night asymmetry would pin down the preferred value to two possibilities on either side of the disfavoured region, and the energy dependence of the night-time rate could in principle resolve the ambiguity.

3. The SuperKamiokande spectral anomaly can be explained in this scenario if Δm^2 takes any of the three or four values corresponding to the local minima in Fig.4.

4. We have shown that increasing SuperKamiokande resolution width by $5 - 10\%$ would also explain the spectral data.

Finally, The Δm^2 region $10^{-3} - 10^{-5}$ eV^2 region can be probed through the $\overline{\nu}_e$ disappearence experiment KAMLAND [19] as well as through the atmospheric neutrino experiments [18]. Note from the above that the region few $\times 10^{-8} - 10^{-9}$ eV^2 is without a smoking-gun signature. Of course, another important test of the scenario for the entire Δm^2 range of interest will be the neutral to charged current ratio at SNO [20]: they should obtain the no-oscillation value. In addition, the BOREXINO [21] and iodine [22] experiments will double check the greater than 50% suppression result distilled from the chlorine experiment.

V. ACKNOWLEDGEMENTS

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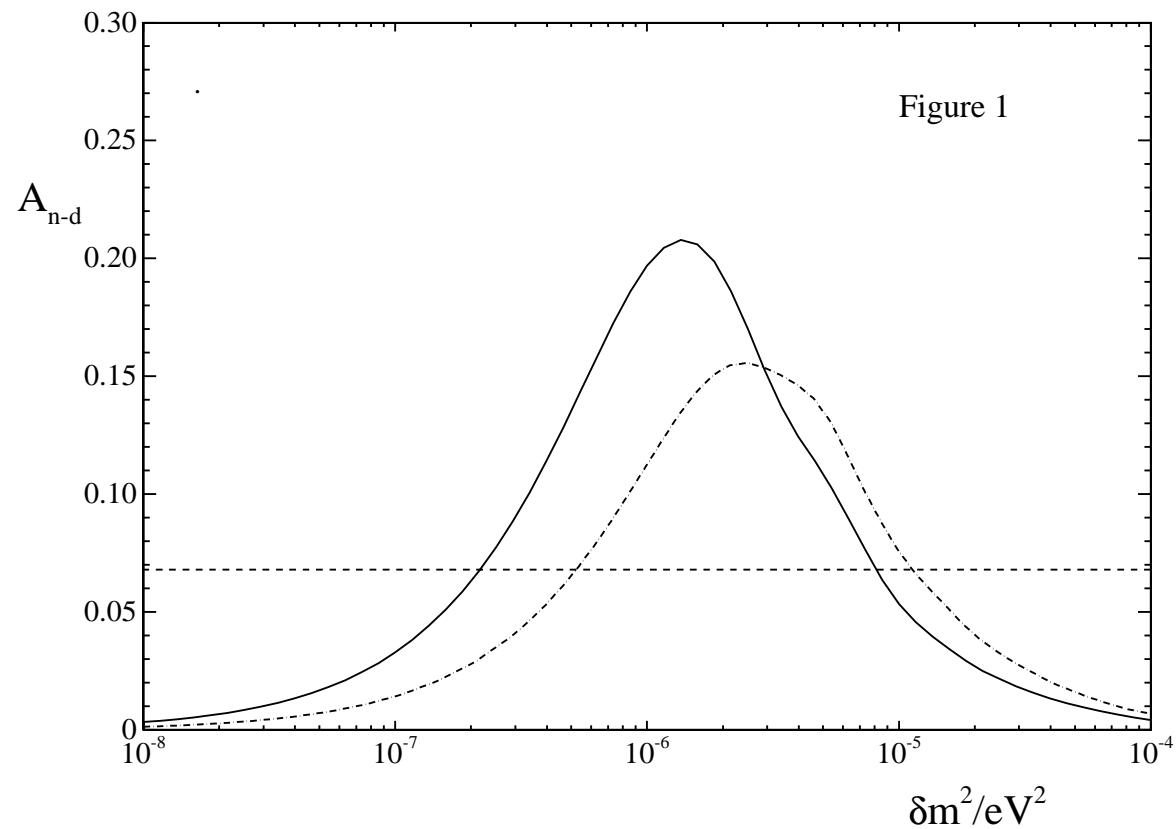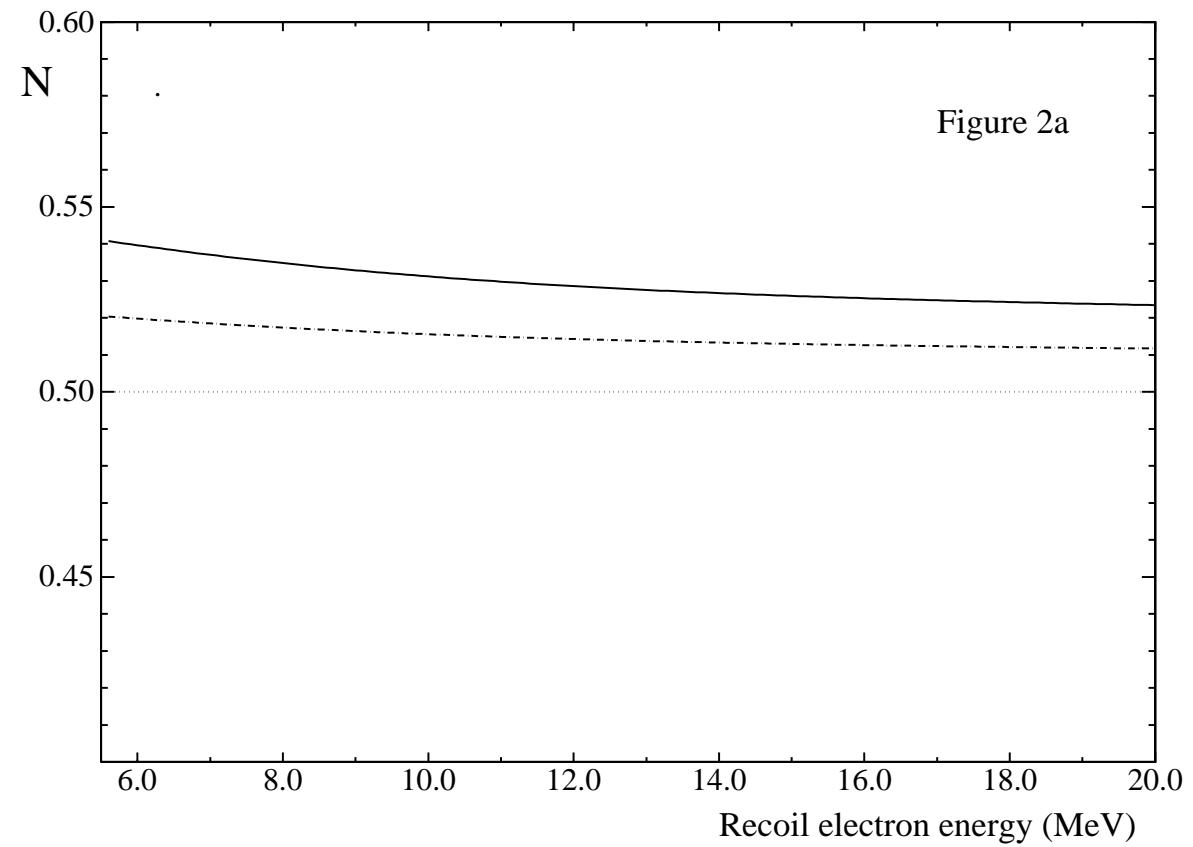## VI. FIGURE CAPTIONS

Figure 1: Night-day asymmetry versus $\Delta m^2/\mathrm{eV}^2$ (solid line) for maximal $\nu_e \to \nu_s$ oscillations. Also shown is the night-day asymetry for maximal $\nu_e \to \nu_\mu$ oscillations for comparison (dashed-dotted line). The horizontal dashed line is the 2σ upper limit.

Figure 2: Predicted recoil electron energy spectrum normalized to the no oscillation expectation. Figure 2a (2b) is for $\Delta m^2/\mathrm{eV}^2 = 10^{-7}$ (10^{-5}). The solid (dotted) line corresponds to the ratio of night-time (day-time) flux per unit energy to the no-oscillation expectation and the dot-dashed line is the average.
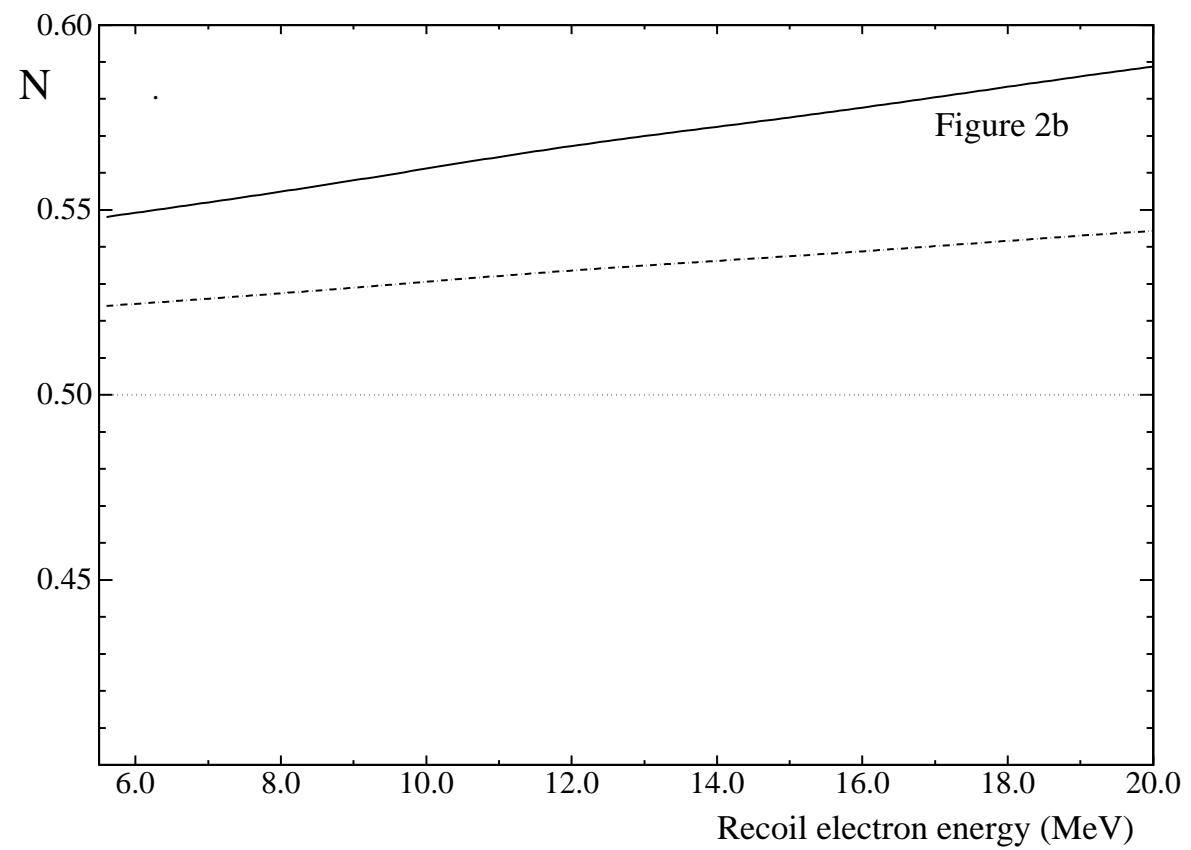
Figure 3: The effect of varying the energy resolution, parameterized by Δ (see text). The figure shows that the spectral anomaly can be explained if $\Delta \simeq 0.50$ MeV instead of the assumed SuperKamiokande value of $\Delta \simeq 0.47$ MeV.

Figure 4: Fit to the SuperKamiokande recoil electron energy spectrum using maximal $\nu_e \to \nu_s$ oscillations.

Figure 1
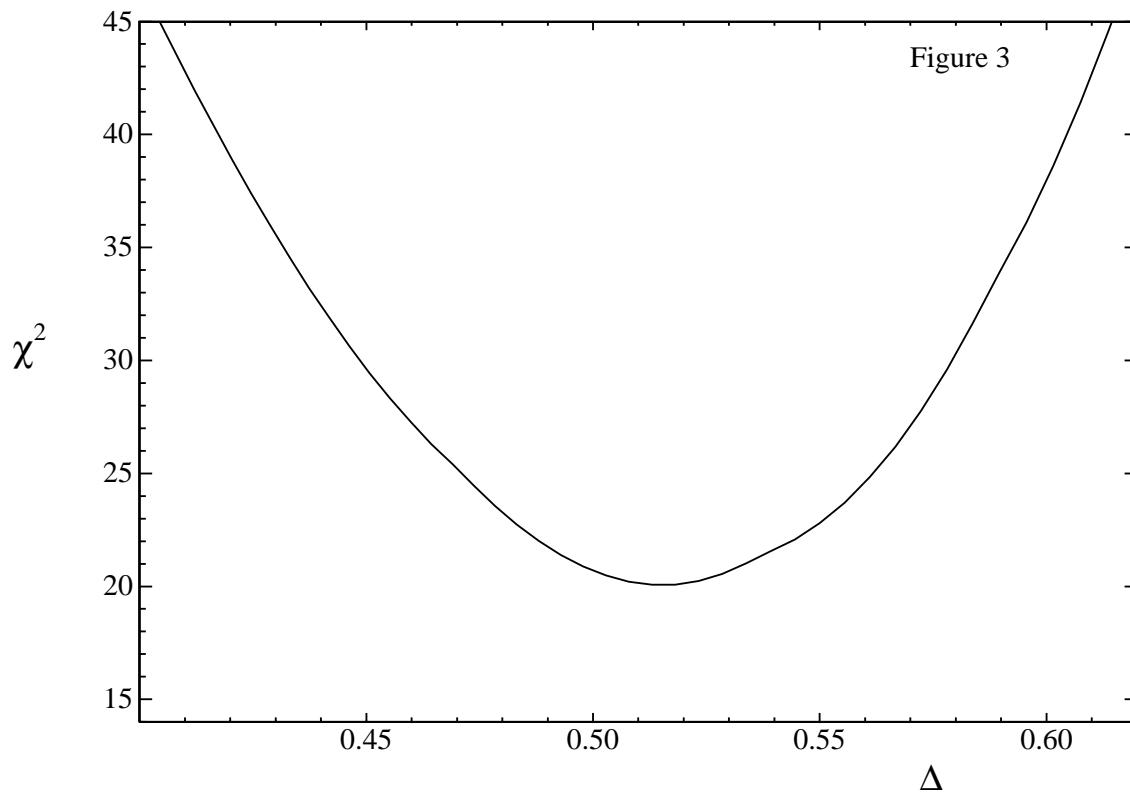
Figure 2a

N

Recoil electron energy (MeV)

N

Figure 2b

Recoil electron energy (MeV)

Figure 3

Figure 4